Cute Subsequences

Let $i_1, \ldots, i_k$ be the indices that maximize the value for each chosen subsequence in the optimal answer. Note that the answer itself is then equal to $a_{i_1} + \ldots + a_{i_k} + max(i_1, \ldots, i_k)$.

This means that if we fix $max(i_1, \ldots, i_k) = x$, we want to maximize the sum of the $k - 1$ elements of the array to the left of the x-th element. To do this, we can move left while maintaining a multiset that stores the current $k - 1$ maximum elements and keeping track of their sum. Then the answer will be the maximum over all x, $x + a_x + sum_x$, where sum_x is the sum of the $k - 1$ maximum elements to the left of x.